

Collection of often used MEMS formulas.

Accelerometers			Material properties		
Mass displacement	$x \approx \frac{m\ddot{x}_f}{k} = \frac{\ddot{x}_f}{\omega_0^2}$	36	Quality factor (scaling)	$f \cdot Q = \text{constant}$	179
Input rererred rms-noise	$\ddot{x}_{\text{rms}} = \sqrt{\frac{\omega_0^2 k_B T}{m}}$	42	Strain	$S = \frac{\Delta L}{L}$	52
Beam optics			Stress	$T = \frac{F}{A}$	52
Beam width	$w(z) \approx \frac{\lambda z}{\pi w_0}$	266	Young's modulus	$E = \frac{T_{XX}}{S_{XX}}$	52
Spreading angle	$\theta_0 \approx \frac{\lambda}{\pi w_0}$	266	Thermal expansion	$S = \alpha \Delta T$	252
Capacitance			Microresonator		
Parallel plate	$C = \epsilon \frac{A}{d-x}$	226	Motional capacitance	$C_m = \frac{\eta^2}{k}$	323
Capacitor charge	$Q = CV$	160	Motional current	$i_{\text{mot}} = \eta \dot{x}$	93
Capacitive actuation			Motional inductance	$L_m = \frac{m}{\eta^2}$	323
General equation	$F_e = -\frac{dW_e}{dx} = \frac{1}{2} V^2 \frac{dC}{dx}$	226	Motional resistance	$R_m = \frac{\sqrt{km}}{Q\eta^2}$	323
Parallel plate	$F_e = \frac{1}{2} \frac{\epsilon A}{(d-x)^2} V^2$	226	MOS transistor		
Comb drive	$F_e = \epsilon N \frac{h}{2d} V^2$	232	1/f-noise	$\overline{v_{1/f}^2} = \frac{K}{C_{ox} W L} \frac{1}{f}$	140
Pull-in displacement	$X_P = \frac{1}{3} d$	228	Input capacitance	$C_{\text{in}} = C_{ox} W L$	131
Pull-in voltage (one sided)	$V_P = \sqrt{\frac{8}{27} \frac{k d^3}{\epsilon_0 A}}$	229	Noise	$\overline{v_{n,\text{in}}^2} = \frac{4k_B T \gamma_c}{g_m}$	141
Pull-in voltage (two sided)	$V_P = \sqrt{\frac{1}{2} \frac{k d^3}{\epsilon_0 A}}$	238	Transconductance	$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}$	131
Transduction factor	$\eta = V_{\text{dc}} \frac{\partial C}{\partial x} \approx V_{\text{dc}} \epsilon \frac{A}{d^2}$	234	Piezoresistance		
Energy			1/f-noise	$\overline{v}_n = \sqrt{\frac{\alpha}{fN}} V$	85
Thermal	$W = \frac{1}{2} k_B T$	20	Gauge factor	$GF = \frac{\Delta R/R}{S} \approx E\pi$	74
Capacitor	$W = \frac{1}{2} C V^2$	20	Piezoresistivity coefficient	$\pi = \frac{\Delta \rho/\rho}{T}$	74
Inductor	$W = \frac{1}{2} L i^2$	20	Resistor resistance	$R = \rho \frac{L}{A}$	74
Spring	$W = \frac{1}{2} k x^2$	24	RF switch		
Inertial	$W = \frac{1}{2} m \dot{x}^2$	23	Capacitive (shunt)	$ S_{21} ^2 = \frac{1}{1+(\omega C Z_0/2)^2}$	295
Fluidic			Series (resistive)	$ S_{21} ^2 = \left \frac{2Z_0}{2Z_0+R} \right ^2$	300
Flow resistance	$R = \frac{8\mu L}{\pi r^4}$	373	Switching time	$t = \sqrt{\frac{27}{2} \frac{V_P}{V_s \omega_0}}$	301
Reynold's number	$Re = \frac{\rho \dot{x} L}{\mu}$	372	Springs		
p_{min} to move bubble	$p_c = \frac{2\gamma}{r}$	374	Hooke's law	$F = kx$	50
Gas damping			Second moment of inertia	$I = \frac{ab^3}{12}$	59
Couette damping	$\gamma = \frac{\mu_{\text{eff}} A}{d}$	192	Cantilever	$k = \frac{3EI}{L^3}$	60
Knudsen number	$K_n = \frac{\lambda}{d_c}$	186	Capacitive	$k_{0e} = -\frac{V^2 C_0}{(d-x_0)^2}$	231
Mean free path	$\lambda = \frac{RT}{\sqrt{2}\pi d_g^2 N_A p}$	185	Guided beam	$k = \frac{12EI}{L^3}$	64
Squeeze film damping	$\gamma_{\text{gas}} = 0.42 \frac{\mu_{\text{eff}} A^2}{d^3}$	188	Rod	$k = \frac{EA}{L}$	58
Gyroscopes			Torsional ($b < a$)	$k_\theta \approx G \frac{ab^3}{3L}$	62
Coriolis force	$\vec{F}_c = -2m\vec{\Omega} \times \vec{\dot{x}}$	347	Springs, parallel	$k_{\text{tot}} = k_1 + k_2$	50
Noise (matched modes)	$\Omega_{\text{rms}} = \frac{1}{2Q_s x_d} \sqrt{\frac{k_B T}{m}}$	360	Springs, series	$\frac{1}{k_{\text{tot}}} = \frac{1}{k_1} + \frac{1}{k_2}$	50
Noise (separated modes)	$\overline{\Omega} = \frac{\sqrt{2} F_{n,s}}{2m_d \omega x_d}$	360	Thermal noise		
Harmonic resonator			Current spectral density	$\overline{i}_n^2 = \frac{4k_B T}{R}$	22
Low-f displacement	$x \approx \frac{F}{k}$	411	Rms displacement	$x_{\text{rms}} = \sqrt{\frac{k_B T}{k}}$	24
Quality factor	$Q = \frac{\omega_0 m}{\gamma} = \frac{\sqrt{km}}{\gamma}$	410	Force spectral density	$\overline{F}_n^2 = 4k_B T \gamma$	24
Resonance frequency	$\omega_0 = \sqrt{\frac{k}{m}}$	410	Rms voltage	$v_{\text{rms}} = \sqrt{\frac{k_B T}{C}}$	20
Resonant displacement	$x = -j \frac{QF}{k}$	411	Voltage spectral density	$\overline{v}_n^2 = 4k_B T R$	21

Note: the number after the formulas corresponds to a page in *Practical MEMS* book.

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